Exploring Optimal Strategies and Variants of "Rugby"

Patrick Hinson and Cody Kim

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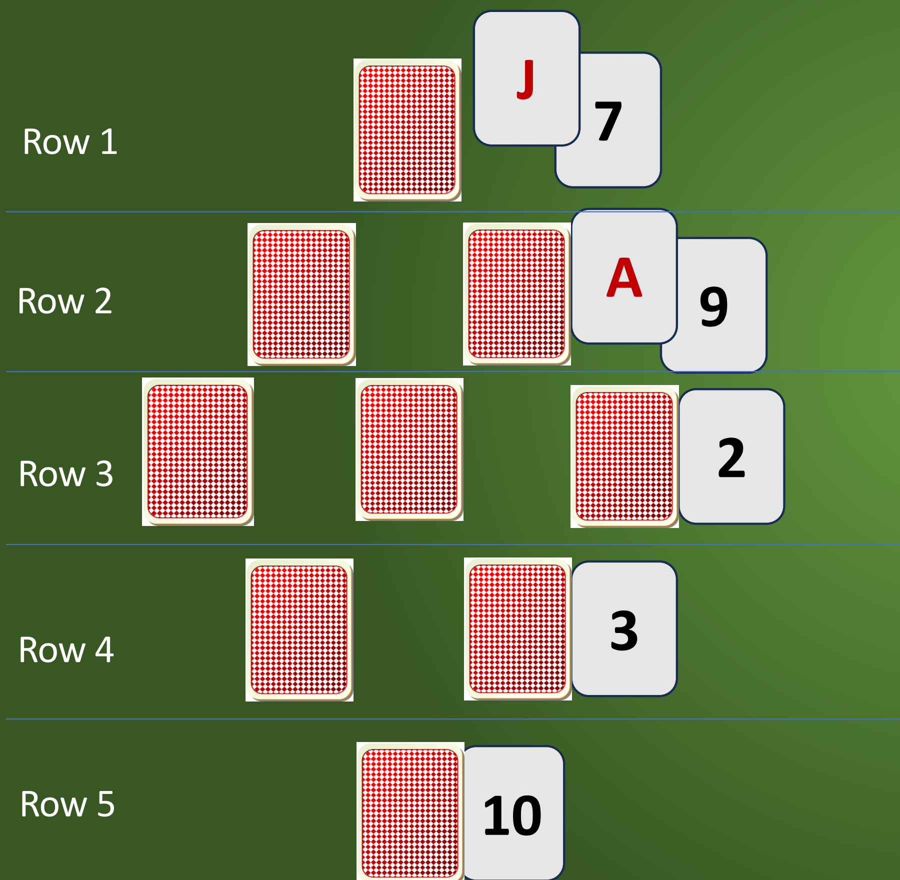
Agenda

- Standard 5-Row Rugby Gameplay and Findings & Theory
- Casino Rugby Gameplay
- 9-Row Rugby Gameplay & Simulations
- Simulation Conclusions

Standard 5-Row Rugby Gameplay



Objective:

The objective of Rugby is to successively flip non-face cards for each row

Start at row 1

If card is face card (J,Q,K,A), deal another card face down from the deck and try again.
If card is non-face card, advance to row 2.

In row 2, select one card and reveal it

If card is face card (J,Q,K,A), deal another card face down from the deck and go back to row 1.
If card is non-face card, advance to row 3.

Game is won when player gets to final row and it is not a face card.
Game is lost when there are no cards to deal after a failed attempt.

Findings

“Two roads diverged in a wood, and I—
I took the one less traveled by,
And that…” makes no difference for Rugby

There is no optimal strategy if playing with a single deck of cards (no reshuffling)

2-10 = 9 non-face
J-A = 4 face

9/13 = 69% non-face cards

>

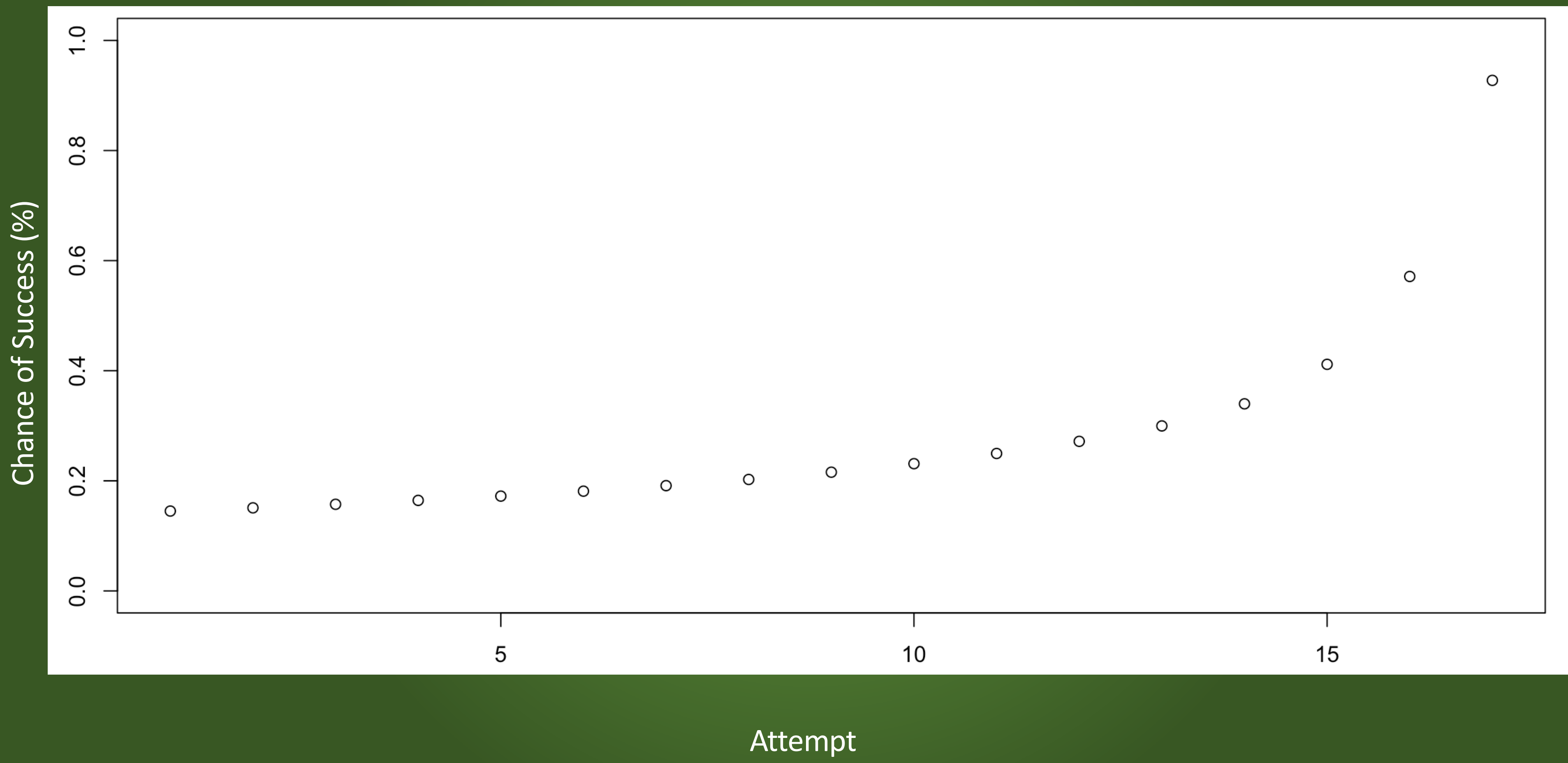
A
K **6**
Q

1/4 = 25% non-face cards

Probability of winning on first attempt:

- Monte Carlo Simulations (n = 10,000, recording # of attempts taken to win)
- $\frac{36}{52} * \frac{35}{51} * \frac{34}{50} * \frac{33}{49} * \frac{32}{48} =$ ~14.5%
- Beyond first turn, probability of success fluctuates depending on how far you got (P(win) is different if you lost on first card compared to losing on last card)
- Monte Carlo Analysis adjusting odds on giving people *n* attempts to win:
  - P(win in 2 or fewer attempts) = ~ 27.4%
  - P(win in 3 or fewer attempts) = ~ 38.8%
  - P(win in 4 or fewer attempts) = ~ 48.9%

Chance of Success by Attempt



Theory

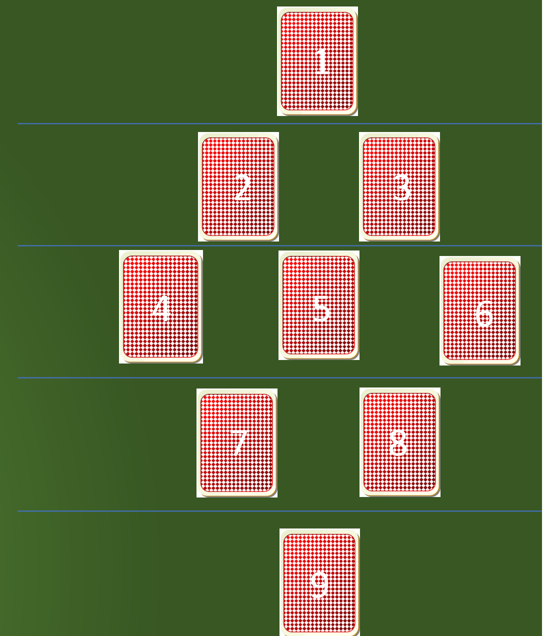
- Basic game (no reshuffling, one deck) Markov chain probabilities
- Each choice has the following probability of success:
- P(Pass) = P(Not Face Card) = $\frac{36 - T}{52 - C}$
- Where T = number of non-face cards revealed, and C is the total number of card revealed (including face cards)

Theory

- Let i = number of turns and j be the j'th card in our game
- Assuming we win on our first try – here's what the Markov chain probability matrix for a success at each turn:

$$P(Turn\ Success) = \begin{bmatrix} \frac{36}{52} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{35}{51} & \frac{35}{51} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{34}{50} & \frac{34}{50} & \frac{34}{50} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{33}{49} & \frac{33}{49} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{32}{48} \end{bmatrix}$$

- $\frac{36}{52} * \frac{35}{51} * \frac{34}{50} * \frac{33}{49} * \frac{32}{48} =$ ~14.5%



Casino Rugby Proposed Payouts

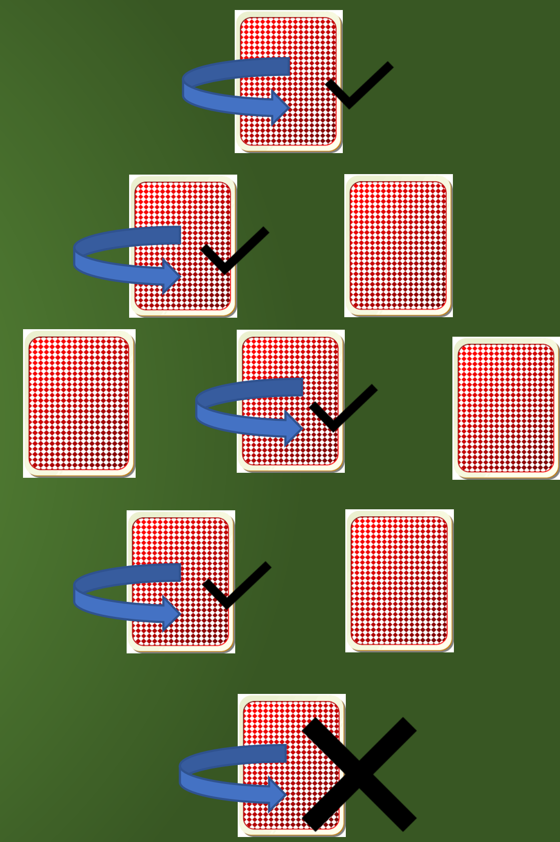
- Beyond first attempt, probability of success fluctuates depending on how far you got
  - P(win) is different if you lost on first card compared to losing on last card

- Monte Carlo Analysis adjusting odds on giving people *n* attempts to win:
  - P(win in 2 or fewer attempts) = ~ 27.4%
  - P(win in 3 or fewer attempts) = ~ 38.8%
  - P(win in 4 or fewer attempts) = ~ 48.9%

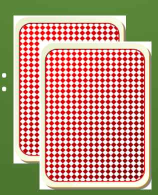
- Bet $100 and:
  - Win in first attempt: $100 -> $600 (break even = ~$688)
  - Win with 2 or fewer attempts: $100 -> $333 (break even = ~$364)
  - Win with 4 or fewer attempts: $100 -> $200 (break even = ~$204)

When shuffling is allowed:

- If you have lost enough times such that no cards remain to reset the board, you can shuffle all flipped cards to recreate the deck and continue playing
- A choice's probability of success depends on which shuffle the card came from
  - Think Monte Hall – probabilities begin to update differently



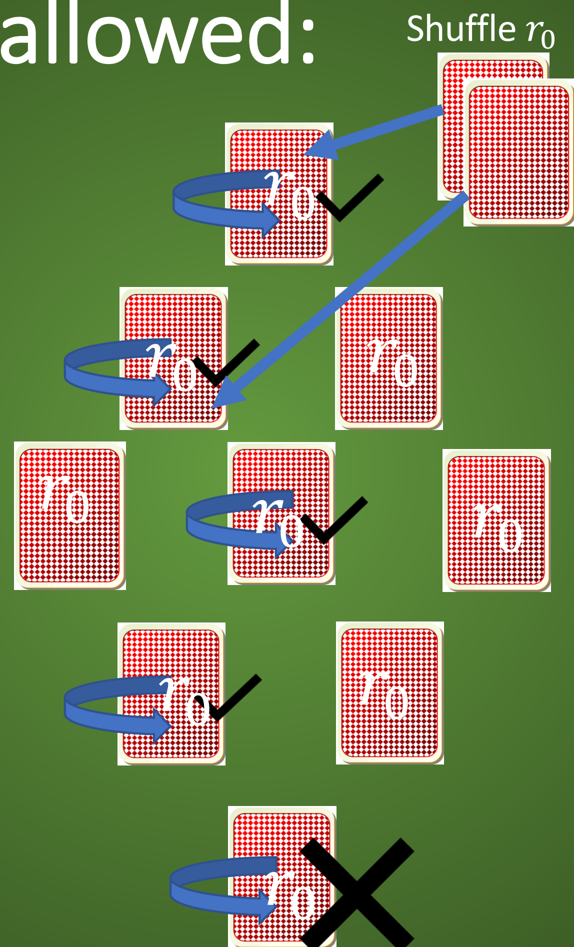
Deck:



Deck doesn't have sufficient cards remaining
Leave all un-flipped cards and reshuffle the rest!

When Shuffling is allowed:

- If we are just starting the game and the cards have not been reshuffled, we can denote this as r_0
- When the player fails enough times such that the deck cannot replace the cards on the board, all the flipped cards on the board are reshuffled, we can denote the cards dealt from this reshuffling this as r_1

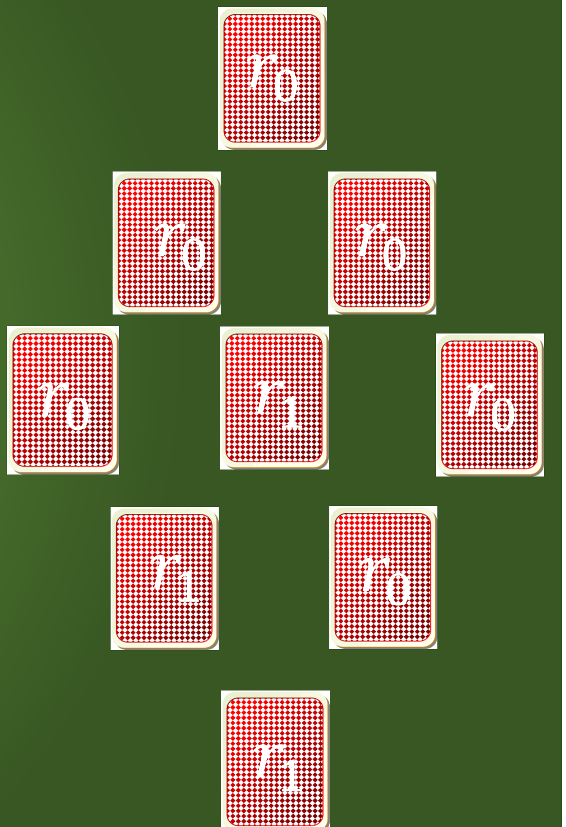


Theory (with shuffling)

- Generally, the formula for each choice becomes

P(Pass) = P(Not Face Card) = $\frac{36 - T_j}{52 - C_j}$

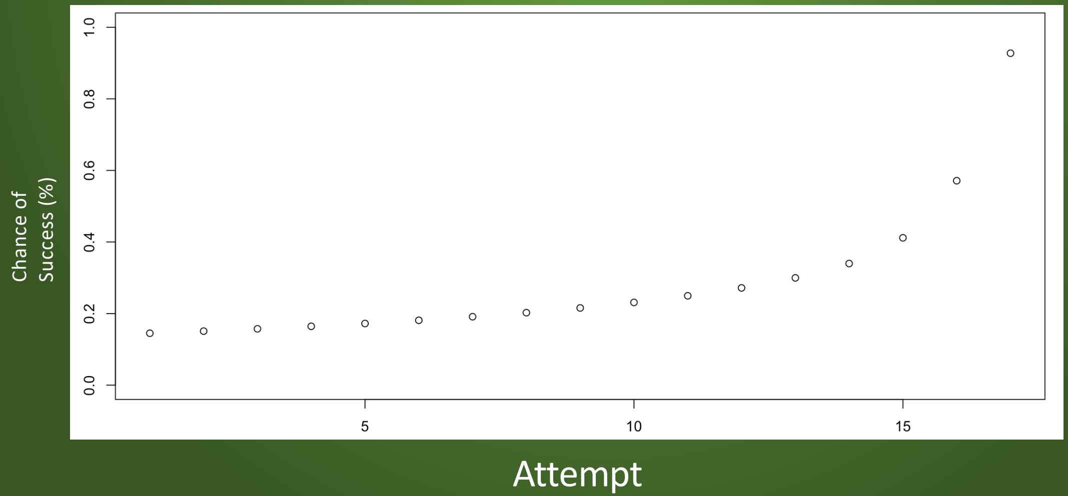
- Where T_j = number of non-face cards revealed so far from the j'th shuffle and C_j is the total number of cards revealed (including face cards) from the j'th shuffle
- This can be derived if you are counting and keeping track of cards



Strategy

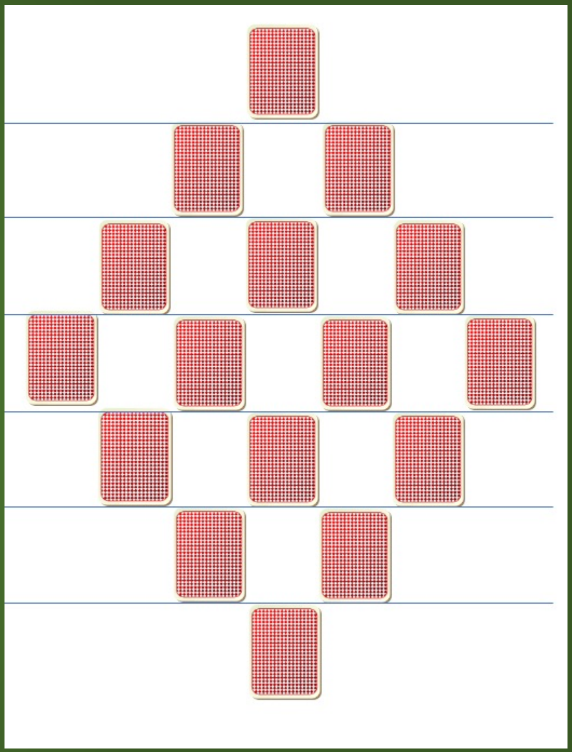
- When all cards on the board belong to the same r_j, strategy does not matter.
  - Choosing randomly vs. going only left will yield the same results.
- When there are more than one r_j in a choice, there is an optimal strategy.
- Shuffles rarely happen with a standard 5 row game. (~1%)

Chance of Success by Attempt (Standard)



Strategy

- Reshuffling is much more frequent with more rows!
- Strategies:
  - Choose randomly each choice
  - Choose the leftmost/rightmost card always
  - Keep track of probabilities, choose option with highest P(success) What happens in a tie?
    - Choose randomly
    - Choose leftmost/rightmost card
- What is best? How do they compare?



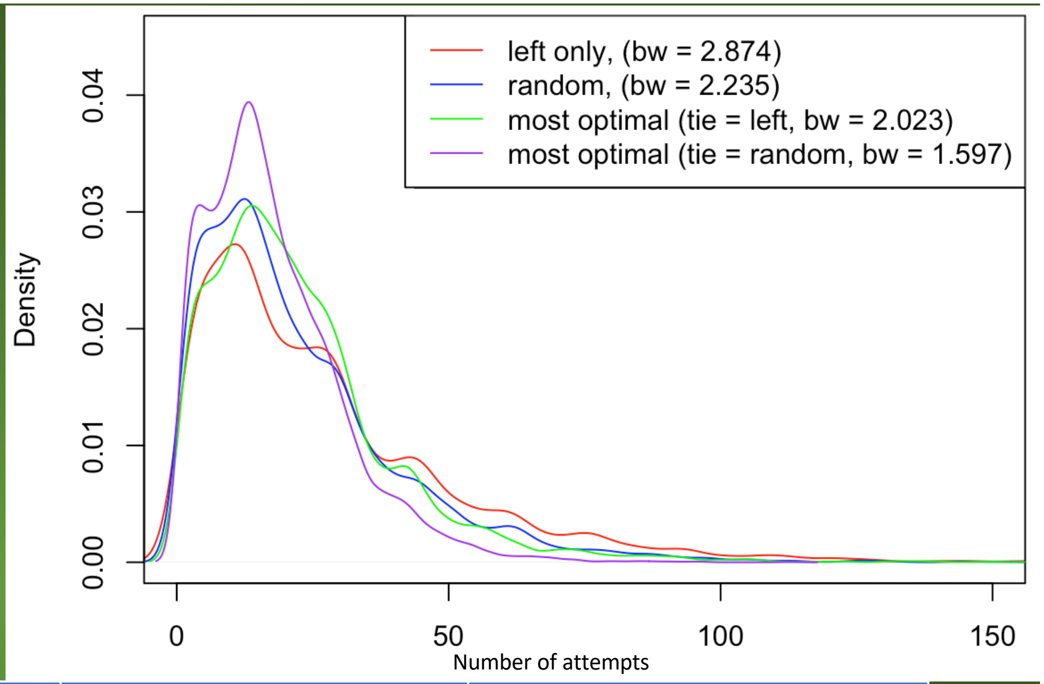
Monte Carlo Simulations

- $\hat{\mu}_{attempts} = \frac{1}{n}\sum_{i=1}^{n} Rugby(strategy)$
- Where Rugby(Strategy) is a random draw/simulation of rugby that follows the given strategy and returns the number of attempts it took to win.
- We estimate the number of attempts a strategy takes to win on a board with 9 rows, 5 cards in the middle, from a standard 52 card playing deck.

Image source: Wikipedia

Results:

Density plot uses R built in density function (gaussian kernel, bandwidths in legend)

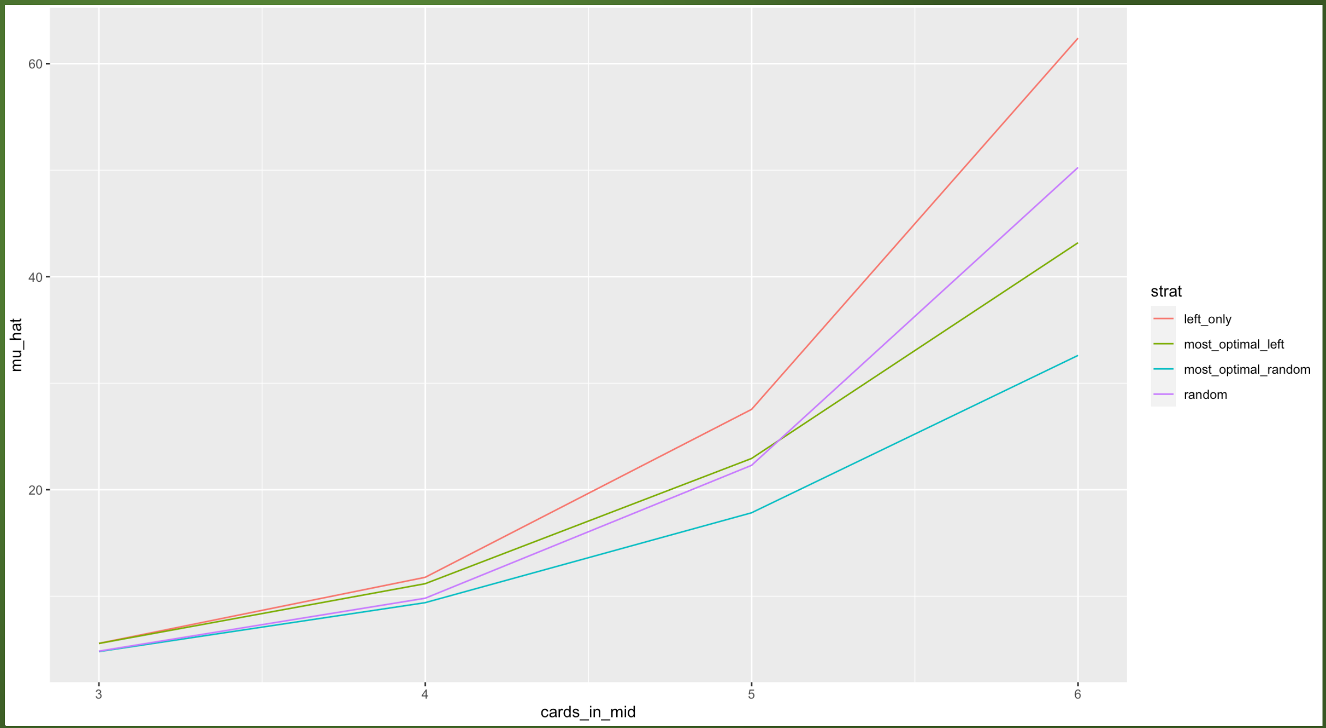


Rank	Strategy	$\hat{\mu}_{attempts}$	$\hat{\sigma}^2_{attempts}$
1	Most optimal choice + Random choice when there is a tie	17.867	150.771
2	Random choices	22.34	346.635
3	Most optimal choice + choose leftmost out of optimal choices	22.42	297.195
4	Left only	27.0085	559.525

Simulation Conclusions

- Randomly choosing is a great strategy!
- Going one path only will sting after a reshuffle and can make a big difference on larger boards
- Potential benefits to taking the road less traveled.

What happens to $\hat{\mu}_{attempts}$ when we play with a bigger board? (n = 10,000)



Thank You!

- Rules for Rugby were drawn from personal experience
- Statistical techniques were drawn from the source material